Implementation of NQS Effects in Large-Signal BJT Models

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***Abstract* —Large-signal implementation of non-quasi-static effects in bipolar transistor is reviewed. An approach strategy is proposed that allows bilateral translation between small- and large-signal equivalent circuits. The approach is illustrated by translating simple small-signal equations commonly used in BJT modeling, as well as more complicated ones proposed in the literature. The present approach extends the state of art by considering arbitrary bias dependence of small-signal time constants.**

I. INTRODUCTION

Compact BJT models often fail as the operating frequency approaches the cutoff frequency and a BJT can not follow external excitations instantaneously. This delay is usually referred to as the non-quasi-static (NQS) effect. As BJTs are increasingly used at a significant fraction of the cutoff frequency, it is important to account for NQS effects in a concise manner so that it can be easily implemented in a compact model.

It is possible to account for NQS effects by using a distributed equivalent circuit [1], [2]. This however leads to complicated models that are difficult to extract from measured data. A practical alternative is to estimate NQS effects analytically and to modify existing compact models to mimic the estimate. For BJTs analytical expressions have been derived to model carrier transport through quasi-neutral base and emitter regions [3]-[7]. However, most of the analysis concentrated on small-signal operations. For large-signal operations transit times and other time constants may vary with the steady-state bias, which makes large-signal equivalent-circuit implementation of NQS effects much more complicated. The implementation proposed so far [3]-[6], [8] are limited to bias-independent time constants.

The purpose of the present work is therefore to propose a simple and systematic approach for implementing NQS effects in a large-signal equivalent-circuit model that accounts for arbitrary bias dependence of time constants. Section II summarizes the status of NQS modeling in BJT and introduces the proposed solution. Section III illustrates the proposed approach. Section IV discusses conversion from current- to voltage-controlled circuit elements, as well as charge conservation issues. Section V concludes the discussion.

II. NQS EFFECTS IN BJT BASE

Small-signal base and collector currents (i_B and i_C) that result from minority carrier transport through the quasi-neutral base have been extensively studied [3]-[7]. Exact small-signal solution is too complex to be used in compact models. Table I summarizes the approximations used in popular compact models as well as proposed in the literature.

Table I
Approximations of i_B and i_C without conductive base current (i_0/β).

Source	i_B	i_C
VBIC, SGP	$i_0 \cdot \tau_F s$	$i_0 \cdot Exp(-\tau_\varphi s)$
MEXTRAM	$i_0 \cdot \tau_F s$	$i_0 \cdot (1-\tau_\varphi s)$
HICUM [9]	$i_0 \cdot \tau_F s\, Exp(-\tau_D s)$	$i_0 \cdot Exp(-\tau_\varphi s)$
[3], [5]	$\frac{i_0 \cdot \tau_F s}{1+\tau_D s}$	$\frac{i_0 \cdot Exp(-\tau_\varphi s)}{1+\tau_D s}$
[6], [7]	$\frac{i_0 \cdot \tau_F s}{1+\tau_D s}$	$\frac{i_0}{1+\tau_D s}$

In Table I, i_0 is the small-signal low-frequency collector current; s is the complex frequency $i \cdot \omega$; τ_F, τ_D and τ_φ are the small-signal time constants. τ_F represents regular quasi-static forward transit time. τ_φ (sometimes together with τ_D) accounts for NQS effects (additional phase shift and magnitude degradation) on i_C. τ_D accounts for NQS effects on i_B.

The first three formulas in Table I are relatively simple. They introduce first-order correction to i_C while partitioning the base charge between emitter and collector [10]. Although this partitioning is normally viewed as an NQS effect, i_B is still regarded as quasi-static. Therefore these simple models (except HICUM) do not introduce any NQS correction to i_B. More accurate approximations are represented in Table 1 by the rational-type formulas with exponential factors sometimes added to improve phase behavior.

Transition to a large-signal equivalent circuit in the simple models is normally performed by integration over the collector current I_C. Small-signal capacitors and trans-

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capacitors thus translate into non-linear charge sources. The problem with this implementation is that the resulting large-signal model does not always revert back to the original small-signal model. The lack of bilateral correlation between small- and large-signal models hampers model extraction since large-signal model is normally obtained from multiple-bias small-signal data. Large-signal equivalent circuits for rational small-signal formulas (that are more accurate) were reported in the literature [3]-[6], [8]. All of them however are limited to bias-independent time constants.

The above discussion provides the motivation for a systematic implementation approach that can bilaterally translate a bias-dependent simple or rational small-signal model into a large-signal equivalent circuit. The proposed approach involves a two-step conversion: First, frequency-domain small-signal equation is converted into time-domain large-signal ordinary differential equation. Second, the ordinary differential equation is implemented as a large-signal equivalent circuit using current-controlled elements (CCEs) such as nonlinear current and charge sources.

Large-signal implementation is not unique and it depends on selected model topology and available circuit elements. Π-topology is adopted in this work since it is widely used in BJT compact models. CCEs are used because they allow concise and intuitive circuit generation.

III. Implementation Examples

Base current in VBIC / SGP models

Using Laplace transformation the small-signal formula for VBIC / SGP base current (1a) can be translated into time domain (1b) as follows:

$$i_B = i_0 \cdot \tau_F s, \tag{1a}$$

$$I_B(t) = \tau_F \frac{d I_0(t)}{dt}, \tag{1b}$$

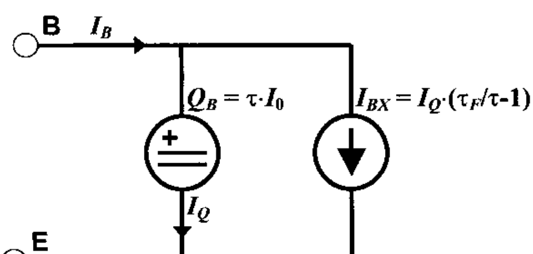

Fig. 1. Large-signal implementation of the base current in a simple small-signal model with τ as a scaling constant.

where I_0 is the large-signal DC collector current, which is considered to be a known external excitation, and I_B is the large-signal base current (without conductive component). Note that although transition form (1a) to (1b) is allowed only if (1a) is linear (i. e., τ_F is bias-independent), the opposite is not true and (1a) follows from (1b) even if τ_F is bias-dependent. Indeed, under a small-signal excitation (1b) can be re-stated as:

$$I_B + \delta I_B(t) = \left(\tau_F + \delta\tau_F(t)\right) \frac{d\left(I_0 + \delta I_0(t)\right)}{dt}, \tag{2}$$

where δ indicates small-signal variations. It can be seen that $\delta\tau_F$ is multiplied by δI_0(t) which makes second order hence can be dropped. Therefore, a large-signal equivalent circuit that obeys (1b) will automatically produce (1a) as its small-signal derivative, even if τ_F is bias-dependent.

Possible large-signal implementation of (1b) is presented in Fig. 1. Its validity can be checked:

$$I_B = I_Q + I_{BX} = \frac{\tau_F}{\tau} I_Q = \frac{\tau_F}{\tau} \frac{d(\tau I_0)}{dt} = \tau_F \frac{d I_0}{dt}. \tag{3}$$

Rational Models

According to the more accurate rational model of Table II without exponential phase factors:

$$i_B = \frac{i_0 \cdot \tau_F s}{1 + \tau_D s}, \quad i_C = \frac{i_0}{1 + \tau_D s}. \tag{4}$$

Although elimination of the phase factors has been questioned recently [11], their marginal improvement in accuracy may not be justified by their complexity. Translating (4) into large-signal ordinary differential equations:

$$\begin{aligned} I_B(t) + \tau_D \frac{d I_B(t)}{dt} &= \tau_F \frac{d I_0(t)}{dt} \\ I_C(t) + \tau_D \frac{d I_C(t)}{dt} &= I_0(t) \end{aligned} . \tag{5}$$

Since the Π-topology has been adopted base and collector currents can be modeled separately resulting in the equivalent circuit in fig. 2.

Special Cases

The rational model implementation in Fig. 2 works for any bias dependence of τ_F and τ_D. It can be simplified in special cases of practical importance:

1) Time constants that depend on the collector current only. In this case Q_{B2} of Fig. 2 can be dropped leaving Q_B and I_{BX} as:

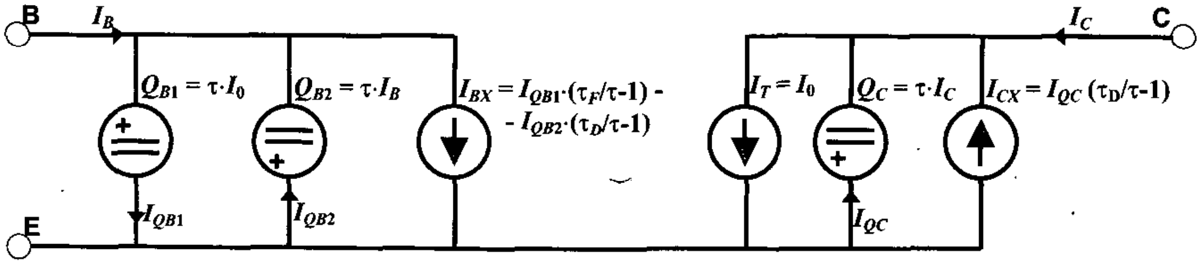

Fig. 2. Large-signal implementation of the rational small-signal model with arbitrary bias dependence of τ_F and τ_D.

$$Q_B = \tau \cdot \left(\int_0^{I_0} \frac{\tau_F}{\tau_D} dI_C - I_B \right), \quad I_{BX} = I_B \left(1 - \frac{\tau}{\tau_D} \right). \tag{6}$$

The fact that (6) obeys (5) can be verified by direct substitution.

2) Bias-independent τ_D. In this case the scaling factor τ in (6) can be set equal to τ_D thus eliminating the current sources I_{BX} and I_{CX}. The base charge source then becomes:

$$Q_B(I_0, I_B) = \int_0^{I_0} \tau_F dI_C - \tau_D I_B . \tag{7}$$

This particular case is important because it conserves base charge as nothing but a charge source remains in the base branch.

3) Bias-independent τ_F and τ_D. In this case, Q_C can be eliminated by setting

$$Q_B = \tau_F I_C - \tau_D I_B, \quad I_T = I_0 - \frac{\tau_D}{\tau_F} I_B . \tag{8}$$

As expected, the corresponding equivalent circuit is equivalent to what was proposed [5], although being more compact due to the choice of CCEs as compared to the voltage-controlled elements.

IV. Discussion

All the equivalent circuits mentioned so far rely on CCEs to facilitate concise and intuitive equivalent circuit generation. Although CCEs are available in modern circuit simulators (e. g., ADS, APLAC), they do not facilitate regular nodal analysis. In particular, current control would cause problems if the model were to be coded in C-language since simulators usually do not provide branch currents for the implementing algorithm. Node voltages on the other hand are available making voltage-controlled elements more preferable. Voltage control can be easily introduced to current-controlled circuit by inserting an arbitrary resistor in controlling current branch. Note that although it appears that three resistors (one each for I_B, I_{QB1} and I_{QB2}, respectively) are required to convert the base side of the circuit in Fig. 2, only two are actually needed since these three currents are not independent of each other, namely: $I_B = (\tau_F I_{QB1} - \tau_D I_{QB2})/\tau$. Similarly, on the collector side only one resistor is needed instead of two since I_C and I_{QC} are also related through $I_C = I_0 - I_{QC} \tau_D / \tau$.

Device physics dictates that charge conservation should hold for the AC component of the base current. However, among the models described by (5)-(8) only (7) and (8), guarantee base charge conservation. In other words, the most complex scenario in terms of bias dependence of τ_F and τ_D that conserves base charge is $\tau_F = \tau_F(I_0)$ and $\tau_D = const.$ It is unclear if there are any less restrictive conditions that could be imposed on bias dependence of τ_F and τ_D in order to make the equivalent circuits in Figs. 1 and 2 conserve base charge. This appears to be an interesting topic for further research.

In order to demonstrate robustness of the implementation technique and compare it with a regular charge-source based approach a small-area (1.4 μm^2) GaAs HBT was simulated using commercially available device simulator [12]. Two models were extracted and implemented using Symbolically Defined Devices in Agilent ADS circuit simulator [13]. The first one was a regular model that utilized charge-partitioning to account for the NQS effects. The second one was implemented according to fig. 2 with a few extra elements added to handle depletion capacitances. Same simulated data were used to extract both models. Shown in fig. 3 is the collector current response of the models together with the results obtained from the device simulator. Transistor was configured as a common-emitter amplifier (V_{CC} = 2V, R_L = 2 kΩ) driven by a 60 ps base voltage pulse (1.2 -> 1.4 V). It can be seen that the technique proposed does better job in matching the device simulator data. No particular convergence problems were noticed as long as the scaling factor τ in fig. 2 was kept on the order of τ_F and τ_D.

IV. Conclusion

In summary, a systematic approach to large-signal implementation of NQS effects in BJT is proposed. It converts small-signal analytical equations into large-signal ordinary differential equations and implement the solution in terms of current-controlled elements. The equivalent-circuit generated using this approach extends previously reported large-signal implementations to include arbitrary bias dependence of time constants. The resulted model is moderately more complex and may not guarantee charge conservation in the base.

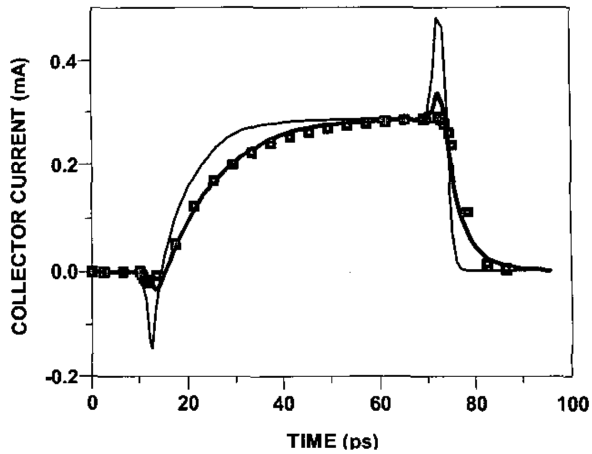

Fig. 3. Collector current response of a 1.4 μm^2 GaAs HBT. The data were obtained from (symbols) the device simulator, (thin line) a regular compact model and (thick line) a compact model implemented using the technique proposed in this work.

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